# The study of quark matter under magnetic fields within NJL models.

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# START SUMMARIZING… NOW!

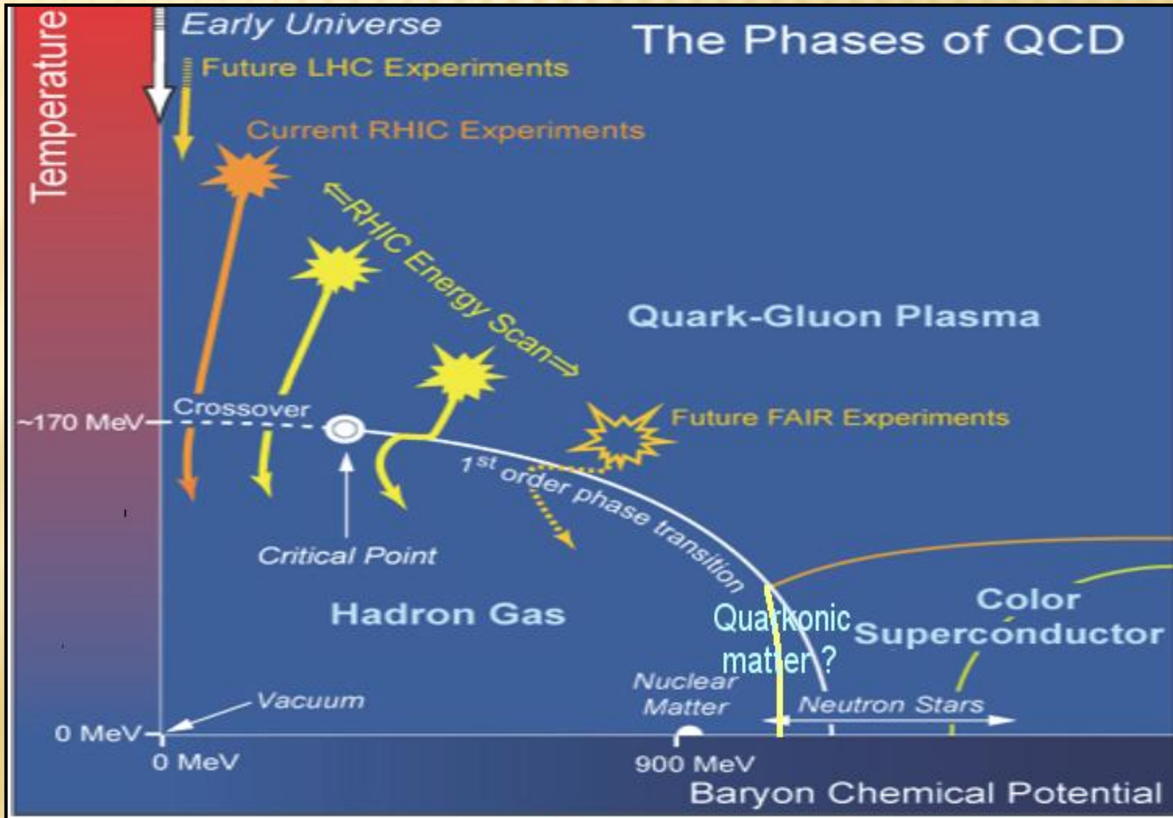
# Motivation:

**Map out the phase diagram of QCD:**
Different states of matter under extreme conditions (T & mu)

Effect of magnetic fields on strongly interacting matter
(order parameter & phase diagrams)

Relevant to the study of heavy ion collisions and compact stars.

Today:
Emphasis on chemical potential axis (T=0)



# The Model – Starting point:

* An effective model for strong interactions in low energy QCD.

* Reproduce dynamical symmetry breaking.

## Nambu Jona Lasinio (NJL) – SU(2) Lagrangian

$$L_{NJL} = \bar{\Psi}(i\partial\!\!\!/ - m_0)\Psi + G\left\{(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\vec{\tau}\gamma_5\Psi)^2\right\}$$

$$\Psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

up and down spinors

G: coupling constant
$m_0$: current quark mass

Interactions:
Gluons are integrated out of the theory and replaced by fermion-fermion point terms.

In the simplest case, scalar-scalar and pseudoscala-isovector terms introduced.

SYMMETRY GROUP: $SU(2)_V \otimes SU(2)_A \otimes U(1)_A \otimes U(1)_V$

# More on the model:

NJL models do not present confinement.

Non renormalizable theory: Regularization prescription for momentum integrals.

**OUR REGULARIZATION:**

Cut-off in 3-momentum:
Additional parameter $\Lambda$

**3 parameters in the model:**

mc: current quark mass
G: interaction strength
$\Lambda$: integration cut off

Parameters adjusted to:

* $m_\pi$ = 138 MeV
* $f_\pi$ = 92.4 MeV
* Quark Dressed Mass: 300 to 600 MeV

NJL + Magnetic field non-trivially dependent on the parameter set.

# Solving the model:

Nambu-Jona-Lasinio Lagrangian cannot be solved exactly.

Ansatz for the ground state.

MEAN FIELD APPROXIMATION

MFA equivalent to the following "linearization"

$$(\bar{\Psi}\Psi)^2 \rightarrow 2\langle\bar{\Psi}\Psi\rangle\bar{\Psi}\Psi \qquad M = m_0 - 2G\langle\bar{\Psi}\Psi\rangle$$

$$(\bar{\Psi}i\vec{\tau}\gamma_5\Psi)^2 \rightarrow 2\langle\bar{\Psi}i\vec{\tau}\gamma_5\Psi\rangle\bar{\Psi}i\vec{\tau}\gamma_5\Psi \qquad \langle\bar{\Psi}i\vec{\tau}\gamma_5\Psi\rangle = 0$$

CHIRAL CONDENSATE ORDER PARAMETER

$$\langle\bar{\Psi}\Psi\rangle = -i\int\frac{d^4p}{(2\pi)^4}TrS(p)$$

Trace calculated over: Flavour, Colour, 4-Momentum

$$M = m_c + 4N_f N_c G\int\frac{d^3p}{(2\pi)^3}\frac{M}{E_p}$$

Gap Equation

Self-Consistent for M

# Extension to finite B & μ:

Chemical Potential → Accounts for the possibility of quark population.

$$n(T,\mu)$$

$$M = m_c + 4N_f N_c G \int \frac{d^3p}{(2\pi)^3} \frac{M}{E_p} \left(1 - n_p(T,\mu) - n_{\bar{p}}(T,\mu)\right)$$

quark/antiquark density:
* fermi distributions for finite T.
* become theta functions for T=0.

Magnetic Field $\vec{B} = B\hat{z}$

$$\vec{D} = \vec{\partial} - ie\vec{A} \qquad \vec{A} = \frac{B}{2}(-y, x, 0)$$

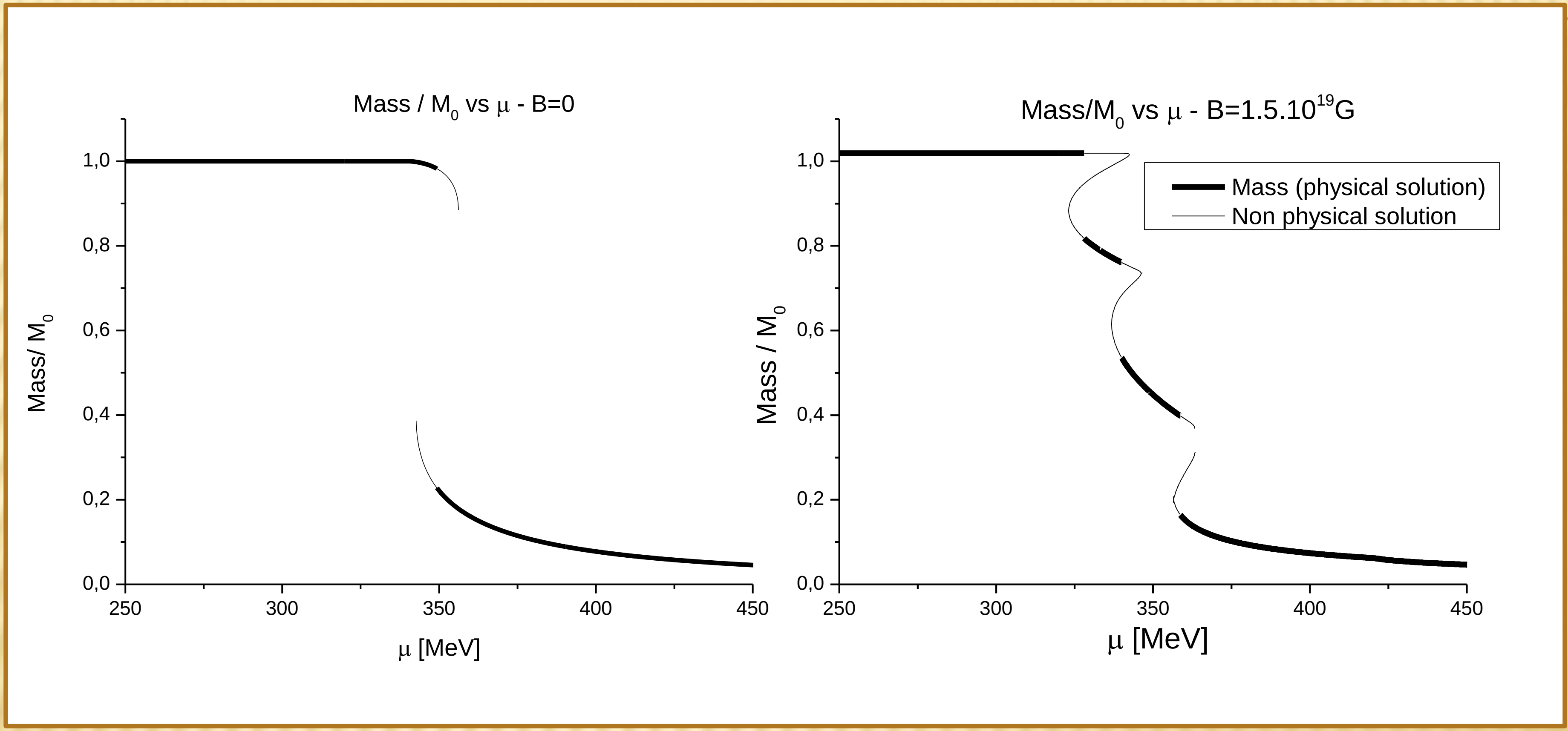
Modified dispersion relation

$$E_p = \sqrt{p^2 + M^2} \longrightarrow E_p^f = \sqrt{p_z^2 + (2n+1-s)|q_f|B + M^2}$$

3- momentum integral replaced by integral in z + sum over Landau levels

$$\int \frac{d^3p}{(2\pi)^3} \longrightarrow \frac{|q_f|B}{2\pi} \sum_{n=0}^{\infty} \int \frac{dp_z}{2\pi}$$

# Mass vs chemical potential:



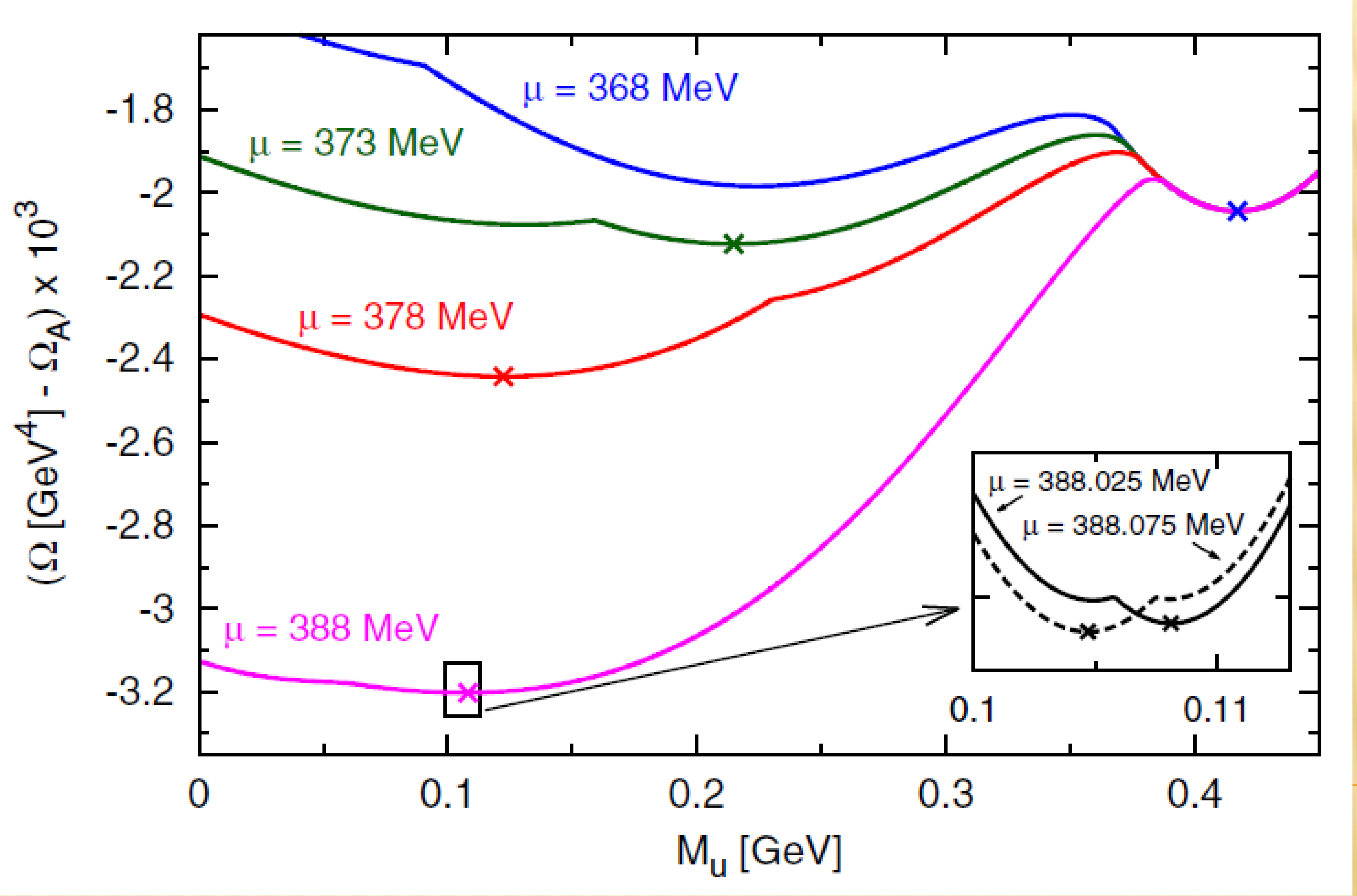

Broken symmetry phase for low μ
Restored symmetry phase for high μ

B=0: single first order transition connecting both phases.

Finite B: transition realized in several steps.

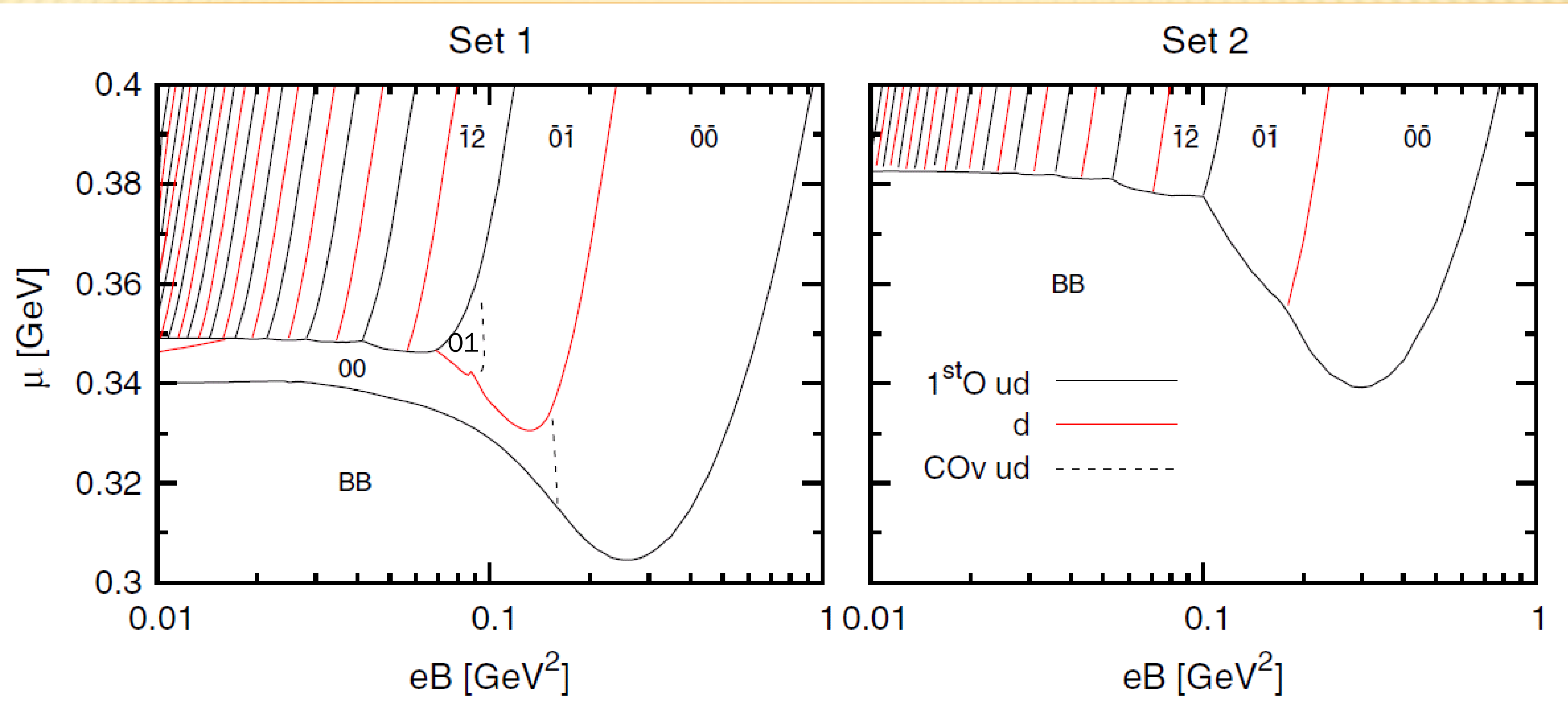
# Thermodynamical potential

## Finite B:



# Phase diagrams μ-eB:

Set 1: $M_0$ = 340MeV
Set 2: $M_0$ = 400MeV




B: vacuum / broken chiral symmetry (no quark population)

n: quark populated up to nth Landau level

The bar distinguishes partially restored symmetry from “fully” restored symmetry

## Summarize the summarized summary of the summary of the ....

How NJL model is solved in the simple possible version of the model

Introduction of finite magnetic field and chemical potential.

Presentation of a few solutions of the M (chiral order parameter) vs μ

A few phase diagrams for different parameter sets

Overview of the classification of possible phases in the μ vs B plane

## Other things I did:

* Analysis of magnetic field of dressed mass in vacuum
(chiral case & finite current mass).
* Regularization procedure for finite magnetic fields.
* Exhaustive analysis of the parameter set dependence.
* Generalize interactions: ‘t Hooft determinant to regulate flavour mixing, introduce vector interacions and diquark pairing.

* Introduce charge neutrality conditions.
* Phase diagram for finite temperature. Inclusion of confinement via Polyakov Loop